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Generalized Finite Sequence of Fuzzy Topographic Topological Mapping

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Abstract: Problem statement: Fuzzy Topographic Topological Mapping (FTTM) was developed to solve the neuromagnetic inverse problem. FTTM consisted of four topological spaces and connected by three homeomorphisms. FTTM 1 and FTTM 2 were developed to present 3-D view of an unbounded single current source and bounded multicurrent sources, respectively. FTTM 1 and FTTM 2 were homeomorphic and this homeomorphism will generate another 14 FTTM. We conjectured if there exist n elements of FTTM, then the numbers of new elements are n^4-n. **Approach:** In this study, the conjecture was proven by viewing FTTMs as sequence and using its geometrical features. **Results:** In the process, several definitions were developed, geometrical and algebraic properties of FTTM were discovered. **Conclusion:** The conjecture was proven and some features of the sequence appear in Pascal Triangle.

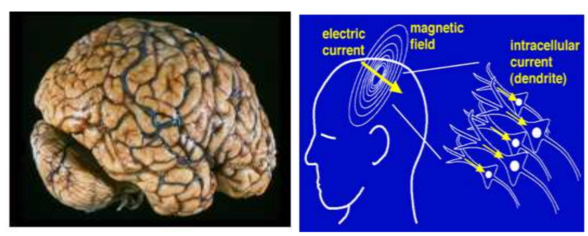
Key words: Fuzzy topographic topological mapping, sequence, Pascal triangle

INTRODUCTION

The human brain (Fig. 1a) is the most important structure in our body. It is also the most complex organized structure known to exist. There are four lobes in both halves of the cortex: Frontal, pariental, temporal and occipital. The outermost layer of the brain is called the cerebral cortex. The cerebral cortex has a total surface area of about 2500 cm^2, folded in a complicated way, so that it fits into the cranial cavity formed by the skull of the brain. There are at least 1010 neurons in the cerebral cortex (Ahmad *et al*., 2008).

These neurons are the active units in a vast signal-handling network. When information is being processed, small currents flow in the neural system and produce a weak magnetic field (Fig. 1b), which can be measured non-invasively by a SQUID (Hamalainen *et al*., 1993) (Superconducting Quantum Interference Device) magnetometer.

Magnetic field readings obtained from SQUID give information for the process to determine location, direction and magnitude of a current source. This is called neuromagnetic inverse problem. Currently there is only a method for solving this problem, namely Bayesian that needs a priori information (data based model) and it is time consuming (Tarantola and Vallete, 1982). On the other hand, FTTM is a novel model for solving neuromagnetic inverse problem (Ahmad *et al*., 2008).



(a) (b)

Fig. 1: (a): Human brain (b): Neuron magnetic field

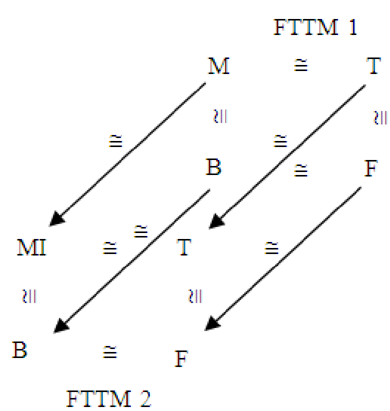


Fig. 2: Homeomorphisms between FTTM 1 and FTTM 2

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It does not need a priori information and it is less time consuming. The development of FTTM has been reported in sequentially. Its mathematical structures, algorithms and its initial performance is reported extensively in (Ahmad *et al*., 2005; 2008). FTTM 1 as well as FTTM 2 are specially designed to have equivalent topological structures between its components. In other words, each component of FTTM 1 and 2 is homeomorphic (Fig. 2) (Ahmad *et al*., 2005).

MATERIALS AND METHODS

Sequence of FTTM: Generally, FTTM can be represented as follows:

$$FTTM = \{(M, B, F, T): M \cong B \cong F \cong T\}$$

$(M', B', F', T') \in FTTM$ means that it satisfy the conditions of M, B, F and T respectively as given in (Ahmad *et al*., 2005) and $M' \cong B'$, $B' \cong F'$ and $F' \cong T'$.

We (Ahmad *et al*., 2005; 2008) proposed the following conjecture with regards in generating FTTM.

Conjecture: If there exist n elements of FTTM illustrated as then the numbers of new elements that can be generated are n^4-n elements.

Even though it seems mathematical induction is most likely can be used to prove the conjecture, unfortunately it didn't work. This is due to the fact that Fig. 3 is a geometrical object while n^4-n is an algebraic expression. Because of this reason, we need to deduce some geometrical features of FTTM in order to relate to the algebraic expression. The idea of deducing such features came from the study of Wu (1997) on Fibonacci Cubes.

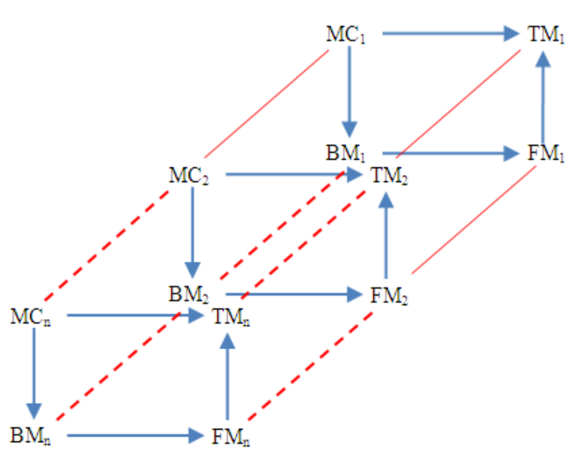

Fig. 3: $FTTM_n$

In order to extract some geometrical features of FTTM, we need to define on how our FTTMs will be arranged (Fig. 4).

Definition 1: Sequence of FTTM.

Let $FTTM_i = (MC_i, BM_I, FM_i, TM_i)$ such that MC_i, BM_I, FM_i, TM_i are topological space with $MC_i \cong BM_i \cong FM_i \cong TM_i$. Set of FTTMi denoted by $FTTM = \{FTTM_i, i = 1, 2, 3, \ldots, n\}$. Sequence of n FTTMi of FTTM is FTTM1, FTTM2, FTTM3,…, FTTMn such that L $MC_i \cong MC_{i+1}$, $BM_i \cong BM_{i+1}$, $FM_i \cong FM_i$, $TM_i \cong TM_i$.

As a start, let us look at $FTTM_1$, $FTTM_2$, $FTTM_3$ and $FTTM_4$ respectively in Fig. 5.

$FTTM_1$ can be viewed generally as a square and without loss of generality we can think MC, BM, FM and TM as vertices and the homeomorphism, i.e., $MC \cong BM$, $BM \cong FM$, $FM \cong TM$ and $MC \cong TM$, as edges. FTTM 1 has 4 vertices and 4 edges.

Similarly FTTM $_2$ contains 8 vertices, 12 edges, 6 faces and 1 cube. Generally a cube is a combination of 2 FTTM.

$FTTM_3$ consists of 12 vertices, 24 edges, 15 faces and 3 cubes.

$FTTM_4$ has 16 vertices, 28 edges, 16 faces and 6 cubes.

Consequently, we can observe some patterns of vertices, edges, faces and cubes emerging from sequences of FTTM as listed in the Table 1.

Definition 2: k-$FTTM_n$.

k-$FTTM_n$ is the k-th FTTM of a sequence of $FTTM_n$ for n≥k. For example, 1-$FTTM_3$, 2-$FTTM_3$ and 3-$FTTM_3$ are given as follows (Fig. 6).

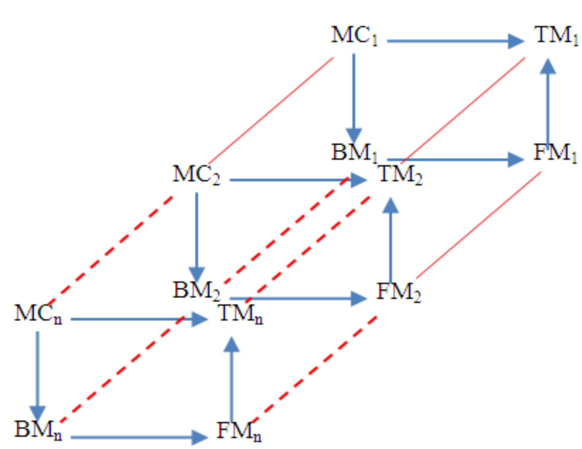

Fig. 4: Sequence of FTTM

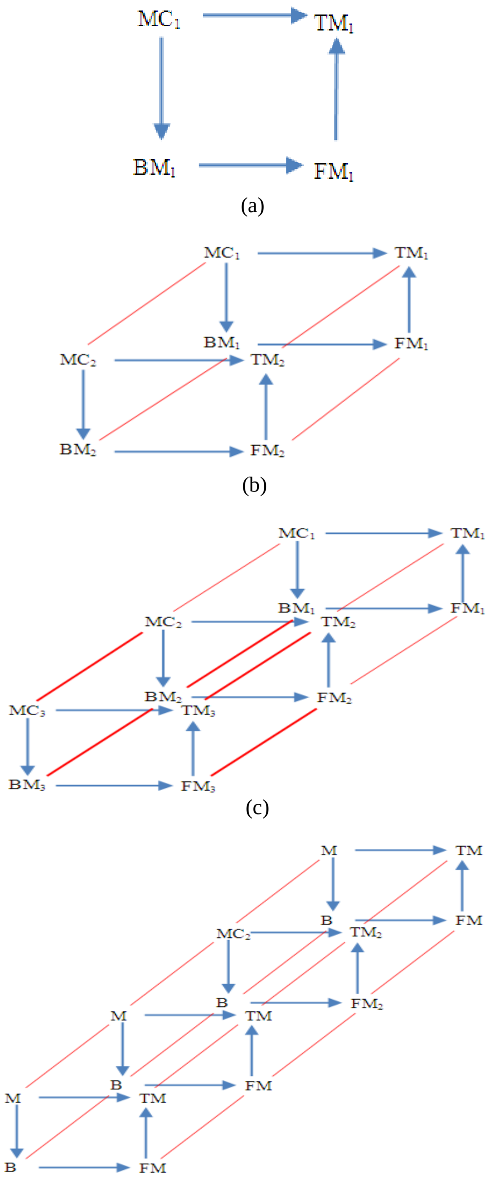

Fig. 5: (a): $FTTM_1$, (b): $FTTM_2$, (c): $FTTM_3$, (d): $FTTM_4$

By defining sequence of FTTMs as given in the Definition 1, considering Table 1 and invoking to arithmetic sequence, we can have the following definition 3-6 which are analogous to the definition of Fibonacci sequence given in (Ahmad *et al*., 2005).

The sequence of vertices begins with the integers 4, 8, 12, 16, 20, 24,… and furthermore it is an arithmetic sequence with the difference between two consecutive terms is 4. We can define formally the sequence of vertices as follows.

Definition 3: Sequence of Vertices of $FTTM_n$.

The sequence of vertices for $FTTM_n$. which is $vFTTM_1$, $vFTTM_2$, $vFTTM_3$,… are given recursively by equation $vFTTM_n = 4n$ for $n \geq 1$.

The sequence of edges begins with the integers 4, 12, 20, 28, 36,… and furthermore it is an arithmetic sequence with the difference between two consecutives term is 8. We can define formally the sequence of edges as follows.

Definition 4: Sequence of Edges of $FTTM_n$.

The sequence of edges for $FTTM_n$ which is $eFTTM_1$, $eFTTM_2$, $eFTTM_3$,… are given recursively by equation $eFTTM_n = 4+(n-1)8$ for $n \geq 1$.

The sequence of faces begins with the integers 1, 6, 11, 16, 21, 26, 31, 36, 41, 46,… and furthermore it is an arithmetic sequence with the difference between two consecutives term is 5. We can define formally the sequence of faces as follows.

Definition 5: Sequence of Faces of $FTTM_n$.

The sequence of faces in $FTTM_n$ is $fFTTM_1$, $fFTTM_2$, $fFTTM_3$,… are defined recursively by the equation $fFTTM_n = 1+(n-1)5$ for $n \geq 1$.

The sequence of cubes begins with the integers 0, 1, 3, 6, 10, 15, 21,… and so on. We can define formally the sequence of cubes as follows.

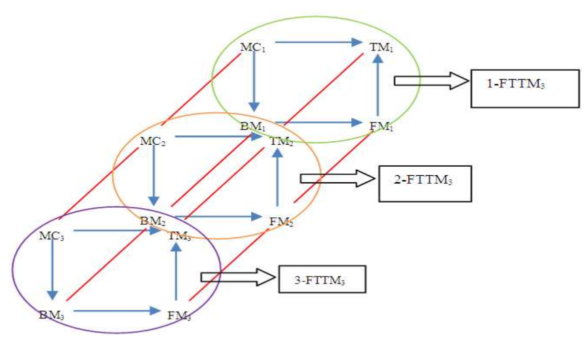

Fig. 6: $1\text{-}FTTM_3$, $2\text{-}FTTM_3$ and $3\text{-}FTTM_3$

Table 1: Vertices, edges, faces and cubes for sequence of FTTM

$FTTM_n$	Vertices	Edges	Faces	Cubes
1	4	4	1	0
2	8	12	6	1
3	12	20	11	3
4	16	28	16	6
5	20	36	21	10
6	24	44	26	15
7	28	52	31	21
8	32	60	36	28
9	36	68	41	36
10	40	76	46	45

Table 2: Sequence of $FTTM_{2/n}$

$FTTM_n$	$FTTM_{2/n}$	14 $FTTM_{2/n}$
$FTTM_1$	0	0
$FTTM_2$	1	14
$FTTM_3$	3	42
$FTTM_4$	6	84
$FTTM_5$	10	140
$FTTM_6$	15	210
$FTTM_7$	21	294
$FTTM_8$	28	392
$FTTM_9$	36	504
$FTTM_{10}$	45	630

Table 3: Sequence of $FTTM_{3/n}$

$FTTM_n$	$FTTM_{3/n}$	36 $FTTM_{3/n}$
$FTTM_1$	0	0
$FTTM_2$	0	0
$FTTM_3$	1	36
$FTTM_4$	4	144
$FTTM_5$	10	360
$FTTM_6$	20	720
$FTTM_7$	35	1260
$FTTM_8$	56	2016
$FTTM_9$	84	3024
$FTTM_{10}$	120	4320

Definition 6: Sequence of Cubes of $FTTM_n$.

The sequence of cubes for $FTTM_n$ which is $FTTM_{2/1}$, $FTTM_{2/2}$, $FTTM_{2/3}$,… are given recursively by equation $FTTM_{2/n} = FTTM_{2/n} +(n-1)$ for $n \geq 1$.

By examining cube in FTTM, we can realize that cube exist from the combination of two terms of FTTM in $FTTM_n$ which in turn will generate another 14 new elements of FTTM. Therefore, Definition 6 can be rewritten as follows.

Definition 7: Sequence of $FTTM_{2/n}$.

$FTTM_{2/n}$ (Table 2) means the number of cubes produced by the combination of any two terms FTTM in $FTTM_n$ with $FTTM_{2/1} = 0$, $FTTM_{2/2} = 1$, $FTTM_{2/3} = 3$, $FTTM_{2/4} = 6$ and in general:

$$FTTM_{2/n} = FTTM_{2/n-1}+(n-1) \text{ for all } n>1$$

Cube in $FTTM_n$ can also be produced from the combination of three terms or three different versions of FTTM in $FTTM_n$. The combination will generate another 36 new elements of FTTM. Follows is the general definition and Table 3 demonstrates sequence of $FTTM_{3/n}$ and its new elements.

Definition 8: Sequence of $FTTM_{3/n}$.

$FTTM_{3/n}$ means the number of cubes produced by the combination of any three terms FTTM in $FTTM_n$ with $FTTM_{3/1}= 0$, $FTTM_{3/2}= 0$. Hence, $FTTM_{3/3} = 1$, $FTTM_{3/4} = 4$, $FTTM_{3/5} = 10$ and in general:

$$FTTM_{3/n} = FTTM_{3/n-1}+FTTM_{2/n-1} \text{ for all } n>1$$

Table 4: Sequence of $FTTM_{4/n}$

$FTTM_n$	$FTTM_{4/n}$	$24FTTM_{4/n}$
$FTTM_1$	0	0
$FTTM_2$	0	0
$FTTM_3$	0	0
$FTTM_4$	1	24
$FTTM_5$	5	120
$FTTM_6$	15	360
$FTTM_7$	35	840
$FTTM_8$	70	1680
$FTTM_9$	126	3024
$FTTM_{10}$	210	5040

We have introduced that two and three terms FTTM in $FTTM_n$ can produce cubes. By extending the number of combination terms of FTTM to four, cube can also be produced and 24 new elements will be generated. Therefore, Definition 9 can be developed as following and Table 4 shows Sequence of $FTTM_{4/n}$ and its new elements.

Definition 9: Sequence of $FTTM_{4/n}$.

$FTTM_{4/n}$ means the number of cubes produced by the combination of any four terms FTTM in $FTTM_n$ with $FTTM_{4/1} = FTTM_{4/2} = FTTM_{4/3}= 0$. Hence, $FTTM_{4/4} = 1$, $FTTM_{4/5} = 5$, $FTTM_{4/6} = 15$ and in general:

$$FTTM_{4/n} = FTTM_{4/n-1}+FTTM_{3/n-1} \text{ for all } n>1$$

RESULTS AND DISCUSSION

It is impossible to continuing develop cube from the combination of five or more terms FTTM. Consequently, the number of generating FTTM in a sequence of $FTTM_n$ is the summation of the three versions of cubes; i.e., $FTTM_{2/n}$, $FTTM_{3/n}$ and $FTTM_{4/n}$. The coefficients for each version are 14, 36 and 24 respectively. The equation which represents the number of new elements of FTTM can be written as below:

$$FTTM_n = 14FTTM_{2/n}+36FTTM_{3/n} = 24FTTM_{4/n} \text{ for } n>1$$

With:

$$FTTM_{2/1}+FTTM_{3/1} = 24FTTM_{4/1} = 0 \tag{1}$$

Table 5 shows our generating $FTTM_n$. The highlighted columns clearly show that calculated generating FTTM using Eq. 1 is equal to Li Yun's conjecture for $FTTM_1$ until $FTTM_{10}$.

Therefore, we need to write $FTTM_{2/n}$, $FTTM_{3/n}$ and $FTTM_{4/n}$ of Eq. 1 into algebraic expressions so that generating $FTTM_n$ can be used to prove the conjecture.

Table 5: Comparison between generating FTTM to Li Yun's conjecture

$FTTM_n$	$14FTTM_{2/n}$	$36FTTM_{3/n}$	$24FTTM_{4/n}$	$14\ FTTM_{2/n}$ + $36\ FTTM_{3/n}$ + $24\ FTTM_{4/n}$	n^4-n
$FTTM_1$	14(0)	36(0)	24(0)	0	0
$FTTM_2$	14(1)	36(0)	24(0)	14	14
$FTTM_3$	14(3)	36(1)	24(0)	78	78
$FTTM_4$	14(6)	36(4)	24(1)	252	252
$FTTM_5$	14(10)	36(10)	24(5)	620	620
$FTTM_6$	14(15)	36(20)	24(15)	1290	1290
$FTTM_7$	14(21)	36(35)	24(35)	2394	2394
$FTTM_8$	14(28)	36(56)	24(70)	4088	4088
$FTTM_9$	14(36)	36(84)	24(126)	6552	6552
$FTTM_{10}$	14(45)	36(120)	24(210)	9990	9990

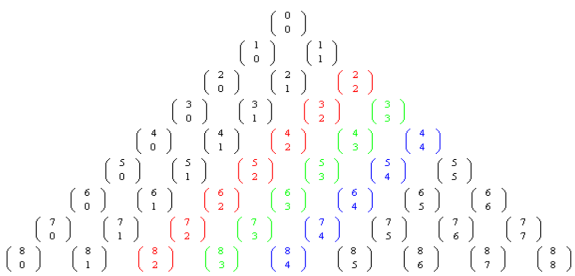

Fig. 7: Pascal triangle

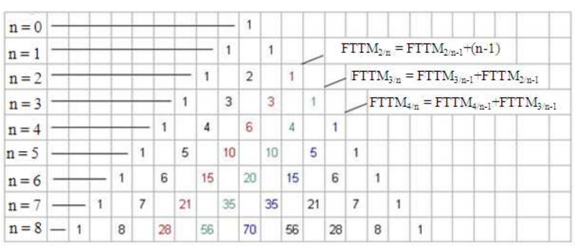

Fig. 8: Sequence of $FTTM_{2/n}$, $FTTM_{3/n}$ and $FTTM_{4/n}$ in Pascal triangle

Interestingly, nonzero sequence of $FTTM_{2/n}$, $FTTM_{3/n}$ and $FTTM_{4/n}$ are presented in the third main diagonal (highlighted by red color), the fourth main diagonal (highlighted by green color) the fifth main diagonal (highlighted by blue color) of Pascal Triangle respectively as shown in Fig. 7.

Each binomial coefficient in Fig. 7 can be calculated and represented as another version of Pascal's Triangle as showed in Fig. 8.

The proof: As mentioned earlier, the problem in proving the conjecture is that the statement is not totally an algebraic statement. By revealing the geometrical features of FTTM and defining their characteristics we produced in Eq. 1. Moreover the third, fourth and fifth of the main diagonal of Pascal's Triangle represent the formulae of sequence of $FTTM_{2/n}$, $FTTM_{3/n}$ and $FTTM_{4/n}$ respectively as illustrated in Fig. 8. In short:

$$FTTM_{2/n} = \binom{n}{2} = \frac{n!}{(2)![n-(2)]!} = \frac{n(n-1)(n-2)!}{2!(n-2)!} = \frac{n(n-1)}{2!} \tag{2}$$

$$FTTM_{3/n} = \binom{n}{3} = \frac{n!}{(3)![n-(3)]!} = \frac{n(n-1)(n-2)(n-3)!}{3!(n-3)!} = \frac{n(n-1)(n-2)}{3!} \tag{3}$$

and

$$FTTM_{4/n} = \binom{n}{4} = \frac{n!}{(4)![n-(4)]!} = \frac{n(n-1)(n-2)(n-3)(n-4)!}{4!(n-4)!} = \frac{n(n-1)(n-2)(n-3)}{4!} \tag{4}$$

By replacing Eq. 2-4 to Eq. 1, the conjecture is finally proven:

$$\begin{aligned}
FTTM_n &= 14\left[\frac{n(n-1)}{2!}\right] + 36\left[\frac{n(n-1)(n-2)}{3!}\right] + 24\left[\frac{n(n-1)(n-2)(n-3)}{4!}\right] \\
&= 7n(n-1) + 6n(n-1)(n-2) + n(n-1)(n-2)(n-3) \\
&= n(n-1)\left[7 + 6(n-2) + (n-2)(n-3)\right] \\
&= n(n-1)\left[7 + 6n - 12 + n^2 - 3n - 2n + 6\right] \\
&= n^2 - n\left[n^2 + n + 1\right] \\
&= n^4 + n^3 + n^2 - n^3 - n^2 - n \\
&= n^4 - n
\end{aligned}$$

CONCLUSION

The aim of this study is to prove the conjecture proposed by (Yun, 2006). We highlighted that the left hand side of the conjecture is a 'geometrical object' in nature while the right hand side is an algebraic expression. Such characteristics of geometrical features that have been produced are sequence of vertices, sequence of edges, sequence of faces and sequence of cubes. These characteristics appeared in Pascal Triangle and were used to prove the conjecture.

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